# Graphing Linear Equations 

This chapter asks you to find solutions to linear equations by graphing. The solution of a linear equation is the set of ordered pairs that form a line on a coordinate graph. Every point on the line is a solution for the equation. One method for graphing the solution is to use a table with $x$ and $y$ values that are solutions for the particular equation. You select a value for $x$ and solve for the $y$ value. But in this chapter, we will focus on the slope and $y$-intercept method.

The slope and $y$-intercept method may require you to change an equation into the slope-intercept form. That is, the equation with two variables must be written in the form $y=\boldsymbol{m} \boldsymbol{x}+\boldsymbol{b}$. Written in this form, the $\boldsymbol{m}$ value is a number that represents the slope of the solution graph and the $\boldsymbol{b}$ is a number that represents the $y$-intercept. The slope of a line is the ratio of the change in the $y$ value over the change in the $x$ value from one point on the solution graph to another. From one point to another, the slope is the rise over the run. The $y$-intercept is the point where the solution graph (line) crosses the $y$-axis. Another way of saying that is: The $y$-intercept is the place where the value of $x$ is 0 .

## Tips for Graphing Linear Equations

- Rewrite the given equation in the form $y=m x+b$.
- Use the $\boldsymbol{b}$ value to determine where the line crosses the $y$-axis.

That is the point $(0, b)$.

- Use the value of $\boldsymbol{m}$ as the slope of the equation. Write the slope as a fraction. If the value of $\boldsymbol{m}$ is a whole number, the slope is the whole number over 1 . The value of $\boldsymbol{m}$ is $=\frac{\text { change in } y}{\text { change in } x}$.
- If the value of $\boldsymbol{m}$ is negative, use a negative sign in only the numerator or the denominator, not both. For example, $\frac{-3}{4}=\frac{-3}{4}=\frac{3}{-4}$.

Graph the following equations using the slope and $y$-intercept method.
171. $y=2 x+3$
172. $y=5 x-2$
173. $y=-2 x+9$
174. $y=\frac{3}{4} x-1$
175. $y=\frac{5}{2} x-3$
176. $y-2 x=4$
177. $y+3 x=-2$
178. $y-\frac{1}{2} x=3 \frac{1}{2}$
179. $2 x+5 y=30$
180. $2 y+4 x=10$
181. $x-3 y=12$
182. $3 x+9 y=-27$
183. $-5 x-y=-\frac{7}{2}$
184. $x=7 y-14$
185. $0=3 x+2 y$
186. $3 x+12 y=-18$
187. $y-0.6 x=-2$
188. $\frac{2}{3} y-\frac{1}{2} x=0$
189. $\frac{5}{6} x-\frac{1}{3} y=2$
190. $7 x=4 y+8$
191. $20 x-15=5 y$
192. $6 y+13 x=12$
193. $0.1 x=0.7 y+1.4$
194. $-34 x+85=17 y$
195. $6 y+27 x=-42$

For the following problems, use the slope $/ y$-intercept method to write an equation that would enable you to draw a graphic solution for each problem.
196. A glider has a $25: 1$ descent ratio when there are no updrafts to raise its altitude. That is, for every 25 feet it moves parallel to the ground, it will lose 1 foot of altitude. Write an equation to represent the glider's descent from an altitude of 250 feet.
197. An Internet service provider charges $\$ 15$ plus $\$ 0.25$ per hour of usage per month. Write an equation that would represent the monthly bill of a user.
198. A scooter rental agency charges $\$ 20$ per day plus $\$ 0.05$ per mile for the rental of a motor scooter. Write an equation to represent the cost of one day's rental.
199. A dive resort rents scuba equipment at a weekly rate of $\$ 150$ per week and charges $\$ 8$ per tank of compressed air used during the week of diving. Write an equation to represent a diver's cost for one week of diving at the resort.
200. A recent backyard bird count showed that one out of every seven birds that visited backyard feeders was a chickadee. Write an equation to represent this ratio.

## Answers

Numerical expressions in parentheses like this [] are operations performed on only part of the original expression. The operations performed within these symbols are intended to show how to evaluate the various terms that make up the entire expression.

Expressions with parentheses that look like this () contain either numerical substitutions or expressions that are part of a numerical expression. Once a single number appears within these parentheses, the parentheses are no longer needed and need not be used the next time the entire expression is written.

When two pair of parentheses appear side by side like this ()(), it means that the expressions within are to be multiplied.

Sometimes parentheses appear within other parentheses in numerical or algebraic expressions. Regardless of what symbol is used, (), \{ \}, or [ ], perform operations in the innermost parentheses first and work outward.

Underlined expressions show the original algebraic expression as an equation with the expression equal to its simplified result.
171. The equation is in the proper
slope $/ y$-intercept form.
$b=3$.
A change in $y$ of 2 and in $x$ of 1 gives the point
$m=2=\frac{2}{1}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,3)$.
$(0+1,3+2)$ or $(1,5)$.


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172. The equation is in the proper
slope $/ y$-intercept form.
$b=-2$.
A change in $y$ of 5 and in $x$ of 1 gives the point
$m=5=\frac{5}{1}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,-2)$.
$(0+1,-2+5)$ or $(1,3)$.

173. The equation is in the proper
slope $/ y$-intercept form.
$b=9$.
A change in $y$ of ${ }^{-2}$ and in $x$ of 1 gives the point


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174. The equation is in the proper
slope $/ y$-intercept form.
$b=-1$.
A change in $y$ of 3 and in $x$ of 4 gives the point
$m=\frac{3}{4}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,-1)$.
$(0+4,-1+3)$ or $(4,2)$.

175. The equation is in the proper
slope/ $y$-intercept form.
$b=-3$.
A change in $y$ of 5 and in $x$ of 2 gives the point
$m=\frac{5}{2}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,-3)$.
$(0+2,-3+5)$ or $(2,2)$.

176. Put the equation in the proper form.

Add $2 x$ to both sides of the
equation.
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=4$.
A change in $y$ of 2 and in $x$ of 1 gives the point
$y+2 x-2 x=2 x+4$
$y=2 x+4$
$m=2=\frac{2}{1}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,4)$.
$(0+1,4+2)$ or $(1,6)$.

177. Put the equation in the proper form.

Subtract $3 x$ from both sides of the equation.
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=-2$.
A change in $y$ of -3 and in $x$ of 1 gives the point
$y+3 x-3 x=-3 x-2$
$y=-3 x-2$
$m=-3=-\frac{3}{1}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,-2)$.
$(0+1,-2-3)$ or $(1,-5)$.

178. Put the equation in the proper form.

Add $\frac{1}{2} x$ to both sides of the
equation.
Simplify the equation.
The equation is in the proper slope/ $y$-intercept form.
$b=3 \frac{1}{2}$.
A change in $y$ of 1 and in $x$ of 2 gives the point
$y+\frac{1}{2} x-\frac{1}{2} x=\frac{1}{2} x+3 \frac{1}{2}$
$y=\frac{1}{2} x+3 \frac{1}{2}$
$m=\frac{1}{2}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $\left(0,3 \frac{1}{2}\right)$.
$\left(0+2,3 \frac{1}{2}+1\right)$ or $\left(2,4 \frac{1}{2}\right)$.

179. Put the equation in the proper form.

Subtract $2 x$ from both sides of the equation.
Simplify the equation.
Divide both sides of the equation by 5 .
Simplify the equation.

The equation is in the proper slope/ $y$-intercept form.
$b=6$.
A change in $y$ of -2 and in $x$ of 5 gives the point
$2 x-2 x+5 y=-2 x+30$
$5 y=-2 x+30$
$\frac{5 y}{5}=\frac{(-2 x+30)}{5}$
$y=\frac{-2 x}{5}+\frac{30}{5}$
$y=\frac{-2}{5 x}+6$
$m=\frac{-2}{5}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,6)$.
$(0+5,6-2)$ or $(5,4)$.

180. Put the equation in the proper form.

Subtract $4 x$ from both sides of
the equation.
Simplify the equation.
Divide both sides of the equation by 2 .
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=5$.
A change in $y$ of -2 and in $x$ of 1 gives the point
$2 y+4 x-4 x=-4 x+10$
$2 y=-4 x+10$
$\frac{2 y}{2}=\frac{(-4 x+10)}{2}$
$y=\frac{-4 x}{2}+\frac{10}{2}$
$y=-2 x+5$
$m=-2=-\frac{2}{1}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,5)$.
$(0+1,5-2)$ or $(1,3)$.

181. Put the equation in the proper form.

Add $3 y$ to both sides
of the equation.
Simplify the equation.
Subtract 12 from both
sides of the equation.
Simplify the equation.
Divide both sides of the equation by 3 .
Simplify the equation.
$\frac{x}{3}=\frac{(1)(x)}{(3)(1)}=\frac{1}{3} \cdot \frac{x}{1}=\frac{1}{3} x$
The equation is equivalent
to the proper form.
The equation is in the proper
slope $/ y$-intercept form.
$b=-4$.
A change in $y$ of 1 and in $x$ of 3 gives the point
$x-3 y+3 y=12+3 y$
$x=12+3 y$
$x-12=12-12+3 y$
$x-12=3 y$
$\frac{(x-12)}{3}=y$
$\frac{x}{3}-\frac{12}{3}=y$
$\frac{x}{3}-4=y$
$\frac{1}{3} x-4=y$
$y=\frac{1}{3} x-4$
$m=\frac{1}{3}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,-4)$.
$(0+3,-4+1)$ or $(3,-3)$.

182. Put the equation in the proper form.

Divide both sides of the equation by 3 .
Simplify the equation.
Subtract $x$ from both sides of the equation.
Simplify the equation.
Divide both sides of the equation by 3 .
Simplify the equation.

The equation is in the proper slope $/ y$-intercept form.
$b=-3$.
A change in $y$ of ${ }^{-1}$ and in $x$ of 3 gives the point

$$
\begin{aligned}
& \frac{(3 x+9 y)}{3}=\frac{-27}{3} \\
& \frac{3 x}{3}+\frac{9 y}{3}=-9 \\
& x+3 y=-9
\end{aligned}
$$

$x-x+3 y=-x-9$
$3 y=-x-9$
$\frac{3 y}{3}=\frac{(-x-9)}{3}$
$y=\frac{-x}{3}-\frac{9}{3}$
$y=\frac{-x}{3}-\frac{-9}{3}$
$y=\frac{-1}{3 x}-3$
$m=-\frac{1}{3}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,-3)$.
$(0+3,-3-1)$ or $(3,-4)$.


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183. Put the equation in the proper form.

Add $5 x$ to both sides of
the equation.
Simplify the equation.
Multiply both sides of the equation by ${ }^{-1}$.
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=\frac{7}{2}=3 \frac{1}{2}$.
A change in $y$ of -5 and in $x$ of 1 gives the point of

$$
\begin{aligned}
& 5 x-5 x-y=5 x-\frac{7}{2} \\
& -y=5 x-\frac{7}{2}
\end{aligned}
$$

$-1(-y)=-1\left(5 x-\frac{7}{2}\right)$
$y=-5 x+\frac{7}{2}$
$m=-5=-\frac{5}{1}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $\left(0,3 \frac{1}{2}\right)$.
$\left(0+1,3 \frac{1}{2}-5\right)$ or $\left(1,-1 \frac{1}{2}\right)$.

184. Put the equation in the proper form.

Add 14 to both sides of
the equation.
Simplify the equation.
Divide both sides of the equation by 7 .
Simplify the equation.
$x+14=7 y+14-14$
$x+14=7 y$
$\frac{(x+14)}{7}=\frac{7 y}{7}$
$\frac{x}{7}+2=y$
$\frac{1}{7} x+2=y$
$y=\frac{1}{7} x+2$
The equation is in the proper slope/ $y$-intercept form. $b=2$.
A change in $y$ of 1 and in $x$ of 7 gives the point
$m=\frac{1}{7}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,2)$.
$(0+7,2+1)$ or $(7,3)$.

185. Put the equation in the proper form.

Subtract $2 y$ from both sides of the equation.
Simplify the equation.
$0-2 y=3 x+2 y-2 y$
Divide both sides of the equation by -2 .
$\frac{-2 y}{-2}=\frac{3 x}{-2}$
Simplify the equation.
$y=\frac{-3}{2} x$
The equation is in the proper slope $/ y$-intercept form.
There is no $b$ showing in the equation, so $b=0$.
A change in $y$ of -3 and in $x$ of 2 gives the point

$$
m=\frac{-3}{2}=\frac{\text { change in } y}{\text { change in } x}
$$

The $y$-intercept is at the point $(0,0)$.
$(0+2,0-3)$ or $(2,-3)$.

186. Put the equation in the proper form.

Divide both sides of the equation by 3 .
Simplify the equation.
Subtract $x$ from both sides of the equation.
Simplify the equation.
Divide both sides of the equation by 4 .
Simplify the equation.

The equation is in the proper slope $/ y$-intercept form.
$b=-\frac{3}{2}$.
A change in $y$ of ${ }^{-1}$ and in $x$ of 4 gives the point

$$
\begin{aligned}
& \frac{(3 x+12 y)}{3}=\frac{-18}{3} \\
& \frac{3 x}{3}+\frac{12 y}{3}=-6 \\
& x+4 y=-6
\end{aligned}
$$

$x-x+4 y=-x-6$
$4 y=-x-6$
$\frac{4 y}{4}=\frac{(-x-6)}{4}$
$y=\frac{-x}{4}-\frac{6}{4}$
$y=-\frac{1}{4} x-1 \frac{1}{2}$
$m=\frac{-1}{4}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $\left(0,-1 \frac{1}{2}\right)$.
$\left(0+4,-1 \frac{1}{2}-1\right)$ or $\left(4,-2 \frac{1}{2}\right)$.

187. Put the equation in the proper form.

Add $0.6 x$ to both sides of the
equation.
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=-2$.
A change in $y$ of 3 and in $x$ of 5 gives the point

$$
\begin{aligned}
& y+0.6 x-0.6 x=0.6 x-2 \\
& y=0.6 x-2
\end{aligned}
$$

$$
m=0.6=\frac{6}{10}=\frac{3}{5}=\frac{\text { change in } y}{\text { change in } x}
$$

$$
\text { The } y \text {-intercept is at the point }(0,-2) \text {. }
$$

$$
(0+5,-2+3) \text { or }(5,1) .
$$


188. Put the equation in the proper form.

Add $\frac{1}{2} x$ to both sides of the equation.
Simplify the equation.
Multiply both sides of the equation by $\frac{3}{2}$.
Simplify the equation.
$\frac{2}{3} y-\frac{1}{2} x+\frac{1}{2} x=0+\frac{1}{2} x$
$\frac{2}{3} y=\frac{1}{2} x$
$\frac{3}{2}\left(\frac{2}{3}\right) y=\frac{3}{2}\left(\frac{1}{2} x\right)$
$1 y=\frac{3}{4} x$
$y=\frac{3}{4} x$
The equation is in the proper slope $/ y$-intercept form.
$b=0$.
A change in $y$ of 3 and in $x$ of 4 gives the point
$m=\frac{3}{4}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,0)$.
$(0+4,0+3)$ or $(4,3)$.

189. Simplify the equation. It would be easier to operate with an equation that doesn't have fractional coefficients. So, if you multiply the whole equation by the lowest common multiple of the denominators, you will have whole numbers with coefficients.
Multiply both sides of the equation by 6 .
Use the distributive property of multiplication.
Simplify the equation.
Subtract $5 x$ from both sides of the equation.
Simplify the equation.
Divide both sides of the equation by -2 .
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=-6$.
A change in $y$ of 5 and in $x$ of 2 gives the point

$$
\begin{aligned}
& 6\left(\frac{5}{6} x-\frac{1}{3} y\right)=6(2) \\
& 6\left(\frac{5}{6} x\right)-6\left(\frac{1}{3} y\right)=6(2) \\
& 5 x-2 y=12 \\
& 5 x-5 x-2 y=-5 x+12 \\
& -2 y=-5 x+12 \\
& \frac{-2 y}{-2}=\frac{-5 x}{-2}+\frac{12}{-2} \\
& y=\frac{5}{2} x-6 \\
& m=\frac{5}{2}=\frac{\text { change in } y}{\text { change in } x} \\
& \text { The } y \text {-intercept is at the point }(0,-6) . \\
& (0+2,-6+5) \text { or }(2,-1) .
\end{aligned}
$$


190. Put the equation in the proper form.

Subtract 8 from both sides
of the equation.
Simplify the equation.
Exchange the terms on each side of the equal sign.
Divide both sides of the equation by 4 .
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=-2$.
A change in $y$ of 7 and in $x$ of 4 gives the point
$7 x-8=4 y+8-8$
$7 x-8=4 y$
$4 y=7 x-8$
$\frac{4 y}{4}=\frac{7 x}{4}-\frac{8}{4}$
$y=\frac{7}{4} x-2$
$m=\frac{7}{4}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,-2)$.
$(0+4,-2+7)$ or $(4,5)$.

191. Exchange the terms on each side
of the equal sign.

$$
5 y=20 x-15
$$

Divide both sides of the equation by 5 .
$\frac{5 y}{5}=\frac{20 x}{5}-\frac{15}{5}$
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=-3$.
A change in $y$ of 4 and in $x$ of 1 gives the point
$(0+1,-3+4)$ or $(1,1)$.

192. Put the equation in the proper form.

Subtract $13 x$ from both sides
of the equation.
Simplify the equation.
Divide both sides of the equation by 6 .
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=2$.
A change in $y$ of -13 and in $x$ of 6 gives the point
$6 y+13 x-13 x=-13 x+12$
$6 y=-13 x+12$
$\frac{6 y}{6}=\frac{-13 x}{6}+\frac{12}{6}$
$y=\frac{-13}{6} x+2$
$m=\frac{-13}{6}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,2)$.
$(0+6,2-13)$ or $(6,-11)$.

193. Once again, if it would be easier for you to operate with whole number coefficients instead of decimals to start, you could multiply the whole equation by 10 .
Multiply both sides of the
equation by 10 .
Simplify the expression.
Subtract 14 from both sides of the equation.
Simplify the equation.
If $a=b$, then $b=a$.
Divide both sides of the equation by 7 .
Simplify the equation.
The equation is in the proper slope/ $y$-intercept form.
$b=-2$.
A change in $y$ of 1 and in $x$ of 7 gives the point
$10(0.1 x)=10(0.7 y+1.4)$
$x=7 y+14$
$x-14=7 y+14-14$
$x-14=7 y$
$7 y=x-14$
$\frac{7 y}{7}=\frac{x}{7}-\frac{14}{7}$
$y=\frac{1}{7} x-2$
$m=\frac{1}{7}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,-2)$.
$(0+7,-2+1)$ or $(7,-1)$.

194. Exchange the terms on each side
of the equal sign.
Divide both sides of the equation by 17 .
Simplify the equation.
The equation is in the proper slope $/ y$-intercept form.
$b=5$.
A change in $y$ of -2 and in $x$ of 1 gives the point

195. Put the equation in the proper form.

Add $-27 x$ to both sides
of the equation.
Simplify the equation.
Divide both sides of the equation by 6 .
Simplify the equation.
Simplify the coefficient of $x$ by a common factor of 3 .
The equation is in the proper slope $/ y$-intercept form.
$b=-7$.
A change in $y$ of 9 and in $x$ of -2 gives the point
$6 y+27 x-27 x=-27 x-42$
$6 y=-27 x-42$
$\frac{6 y}{6}=\frac{-27 x}{6}-\frac{42}{6}$
$y=\frac{-27}{6} x-7$
$y=\frac{-9}{2} x-7$
$m=\frac{-9}{2}=\frac{9}{-2}=\frac{\text { change in } y}{\text { change in } x}$
The $y$-intercept is at the point $(0,-7)$.
$(0-2,-7+9)$ or $(-2,2)$.

196. Let $x=$ horizontal movement. Forward is in the positive direction.

Let $y=$ vertical movement. Ascending is in the positive direction.
Descending is in the negative.
The change in position of the glider is described by the slope.
The change in $y$ is ${ }^{-1}$ for every change in $x$ of +25 .

Slope $=\frac{\text { change in } y}{\text { change in } x}=\frac{-1}{25}=m$

The starting position for the purposes of this graphic solution is at an altitude of 250 ft or ${ }^{+250}$. So:
$b=250$
Using the standard form $y=m x+b$, you substitute the given values into the formula.
A graph of this equation would have a slope of $-\frac{1}{25}$ and the $y$-intercept would be at
$y=\frac{-1}{25} x+250$
$(0,250)$
197. Let $y=$ the amount of a monthly bill.

Let $x=$ the hours of Internet use for the month.
The costs for the month will equal $\$ 15$ plus the $\$ .25$ times the number of hours of use.
Written as an equation, this information would be as follows:
A graph of this equation would have a slope of 0.25 or
The $y$-intercept would be at
$\frac{y=0.25 x+15}{\frac{25}{100}=\frac{1}{4}}$
$(0,15)$
198. Let $y=$ the cost of a scooter rental for one day.

Let $x=$ the number of miles driven in one day.
The problem tells us that the cost would be equal to the daily charges plus the 0.05 times the number of miles driven.
Written as an equation, this would be
The graph would have a $y$-intercept at $(0,20)$ and the slope would be

$$
y=0.05 x+20
$$

$$
\frac{5}{100}=\frac{1}{20}
$$

199. Let $y=$ the total cost for equipment.

Let $x=$ the number of tanks used during the week.
The problem tells us that the cost would be equal to the weekly charge for gear rental plus 8 times the number of tanks used.
A formula that would represent this information would be:
$y=8 x+150$
The $y$-intercept would be at $(0,150)$ and the slope $=8=\frac{8}{1}$.
200. Let $y=$ the number of birds that visited a backyard feeder.

Let $x=$ the number of chickadees that visited the feeder.
An equation that represents the statement would be:
$y=7 x$
The $y$-intercept is $(0,0)$ and the slope $=7=\frac{7}{1}$.

